Ceng124
Discrete Structures

2018-2019 Spring Semester

Course Information

- Instructor: Assist. Prof. Dr. Sibel Tarıyan Özyer
- Department: Computer Engineeering
- Office: L216 Phone: 2331355
- Email: tariyan@cankaya.edu.tr
- Website: ceng124.cankaya.edu.tr

Course Goals

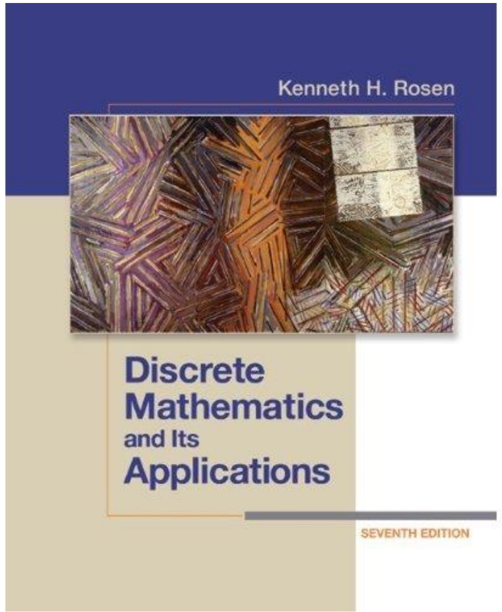
- Mathematical reasoning
  - Logic
- Discrete structures
  - Sets, sequences, functions, relations, graphs, trees
- Algorithmic reasoning
  - Specifications and verifications
- Boolean algebra
- Machine and languages

Course Description

- Logic, sets, relations and functions, application to data structure and graph representations, partial ordered sets, trees, algebraic structures, Boolean algebra, introduction to grammars, machines and languages, error correcting codes.

Textbook

- Discrete Mathematics and Its Applications, Kenneth Rosen Seventh Edition, 2012, Mc GrawHill.



Grading

- %30 Midterm Examination
- %30 Quizes and Homeworks
- %40 Final Examination

Prerequisite

Ceng442 Programming Language Concepts

Ceng491 Formal Languages and Automata

||

V

Ceng124 Discrete Structures

Topics

- Logic
- Sets
- Functions
- Relations
- Graph
- Trees
- Boolean algebra
- Machine and languages

1.1 Propositional logic

- Understand and construct correct mathematical arguments
- Give precise meaning to mathematical statements
- Rules are used to distinguish between valid (true) and invalid arguments
- Used in numerous applications: circuit design, programs, verification of correctness of programs, artificial intelligence, etc.

Proposition

- A declarative sentence that is either true or false, but not both
  - Ankara, is the capital of Turkey
  - Bolu is adjacent to Ankara
  - 1+1=2
  - 2+2=5
  - What time is it?

Logical operators

- Negation operator
- Conjunction (and, ^)
- Disjunction (or v)
- Conditional statement (if then, →)
- Biconditional statement (if and only if ←→)
- Exclusive Or (XOR)

Negation

Let p be a proposition. The *negation of* p, denoted by $\neg p$ (also denoted by $\overline{p}$), is the statement

"It is not the case that p."

The proposition $\neg p$ is read "not p." The truth value of the negation of p, $\neg p$, is the opposite of the truth value of p.

TABLE 1 The Truth Table for the Negation of a Proposition.

p	$\neg p$
T	F
F	T

Example

- “Today is Friday”
  - It is not the case that today is Friday
  - Today is not Friday
- “Michaels PC runs Linux”
  - It is not the case that Michaels PC runs Linux
  - Michaels PC does not run Linux

Conjunction

Let p and q be propositions. The *conjunction* of p and q, denoted by $p \wedge q$, is the proposition "p and q." The conjunction $p \wedge q$ is true when both p and q are true and is false otherwise.

TABLE 2 The Truth Table for the Conjunction of Two Propositions.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Example

- p: “Today is Friday”, q: “It is raining today”
- p ∧ q “Today is Friday and it is raining today”
  - true: on rainy Fridays
  - false otherwise:
    - Any day that is not a Friday
    - Fridays when it does not rain

Disjunction

Let p and q be propositions. The *disjunction* of p and q, denoted by $p \vee q$, is the proposition "p or q." The disjunction $p \vee q$ is false when both p and q are false and is true otherwise.

TABLE 3 **The Truth Table for the Disjunction of Two Propositions.**

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Example

- p ∨ q: "Today is Friday or it is raining today"
  - True:
    - Today is Friday
    - It is raining today
    - It is a rainy Friday
  - False
    - Today is not Friday and it does not rain

Exclusive Or

Let p and q be propositions. The *exclusive or* of p and q, denoted by $p \oplus q$, is the proposition that is true when exactly one of p and q is true and is false otherwise.

TABLE 4 The Truth Table for the Exclusive Or of Two Propositions.

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

Conditional Statement

Let p and q be propositions. The *conditional statement* $p \rightarrow q$ is the proposition "if p, then q." The conditional statement $p \rightarrow q$ is false when p is true and q is false, and true otherwise. In the conditional statement $p \rightarrow q$, p is called the *hypothesis* (or *antecedent* or *premise*) and q is called the *conclusion* (or *consequence*).

TABLE 5 The Truth Table for the Conditional Statement $p \rightarrow q$.

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Conditional statement p→q

if p, then q if p, q p is sufficient for q q if p q when p a necessary condition for p is q q is unless ┐ p	p implies q p only if q a sufficient condition for q is p q whenever p q is necessary for p q follows from p

Example
If I am elected I will lower taxes.

TABLE 5 The Truth Table for the Conditional Statement $p \rightarrow q$.

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

p→q

- p only if q:
  - p cannot be true when q is not true
  - The statement is false if p is true but q is false
  - When p is false, q may be either true or false
  - Not to use “q only if p” to express p→q
- q unless ┐ p
  - If ┐ p is false, then q must be true
  - The statement is false when p is true but q is false, but the statement is true otherwise

TABLE 5 The Truth Table for the Conditional Statement $p \to q$.

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

Example

- "If today is Friday, then 2+3=6"
  - The statement is true every day except Friday even though 2+3=6 is false

TABLE 5 The Truth Table for the Conditional Statement $p \rightarrow q$.

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Biconditional Statement

Let p and q be propositions. The *biconditional statement* $p \leftrightarrow q$ is the proposition "p if and only if q." The biconditional statement $p \leftrightarrow q$ is true when p and q have the same truth values, and is false otherwise. Biconditional statements are also called *bi-implications*.

TABLE 6 The Truth Table for the Biconditional $p \leftrightarrow q$.

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Example

- P: “you can take the flight”, q: “you buy a ticket”
- P ←→ q: “You can take the flight if and only if you buy a ticket”
  - This statement is true
    - If you buy a ticket and take the flight
    - If you do not buy a ticket and you cannot take the flight
- Note that P ←→ q has exactly the same truth values as (p->q) ^ (q->p)

Truth Table of Compound Preposition

TABLE 7 The Truth Table of $(p \vee \neg q) \rightarrow (p \wedge q)$.

p	q	$\neg q$	$p \vee \neg q$	$p \wedge q$	$(p \vee \neg q) \rightarrow (p \wedge q)$
T	T	F	T	T	T
T	F	T	T	F	F
F	T	F	F	F	T
F	F	T	T	F	F

Precedence of Logical Operators

TABLE 8
Precedence of Logical Operators.

Operator	*Precedence*
$\neg$	1
$\wedge$	2
$\vee$	3
$\rightarrow$	4
$\leftrightarrow$	5

Bit Operations

TABLE 9 Table for the Bit Operators *OR*, *AND*, and *XOR*.

x	y	$x \vee y$	$x \wedge y$	$x \oplus y$
0	0	0	0	0
0	1	1	0	1
1	0	1	0	1
1	1	1	1	0

Example

Find the bitwise OR, AND, XOR of the bit strings 01 1011 0110 and 11 0001 1101

Solution:

01 1011 0110	
11 0001 1101	
11 1011 1111	bitwise *OR*
01 0001 0100	bitwise *AND*
10 1010 1011	bitwise *XOR*