Ceng124
Discrete Structures

2018-2019 Spring Semester

Course Information

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Course Goals

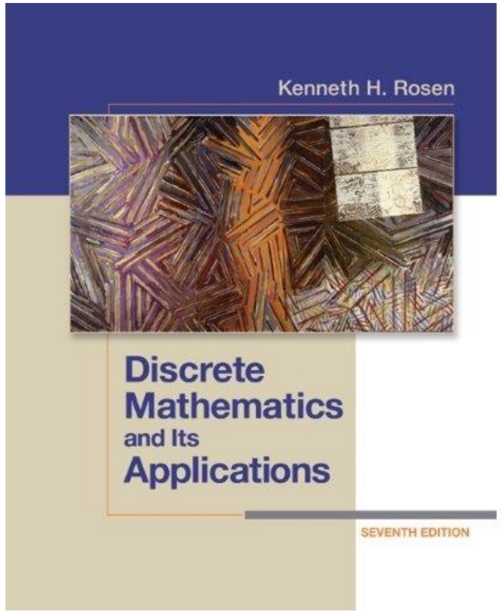
- Mathematical reasoning
  - Logic, proof
- Discrete structures
  - Sets, sequences, functions, relations, graphs, trees
- Algorithmic reasoning
  - Specifications and verifications
- Boolean algebra
- Machine and languages

Course Description

- Logic and proof, sets, relations and functions, application to data structure and graph representations, partial ordered sets, trees, algebraic structures, Boolean algebra, introduction to grammars, machines and languages, error correcting codes.

Textbook

- Discrete Mathematics and Its Applications, Kenneth Rosen Seventh Edition, 2012, Mc GrawHill.



Grading

- %30 Midterm Examination
- %30 Quizes and Homeworks
- %40 Final Examination

Prerequisite

Ceng442 Programming Language Concepts

Ceng491 Formal Languages and Automata

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Ceng124 Discrete Structures

Topics

- Logic
- Proof
- Sets
- Functions
- Relations
- Graph
- Trees
- Boolean algebra
- Machine and languages

1.1 Propositional logic

- Understand and construct correct mathematical arguments
- Give precise meaning to mathematical statements
- Rules are used to distinguish between valid (true) and invalid arguments
- Used in numerous applications: circuit design, programs, verification of correctness of programs, artificial intelligence, etc.

Proposition

- A declarative sentence that is either true or false, but not both
  - Ankara, is the capital of Turkey
  - Bolu is adjacent to Ankara
  - 1+1=2
  - 2+2=5
  - What time is it?

Logical operators

- Negation operator
- Conjunction (and, ^)
- Disjunction (or v)
- Conditional statement (if then, →)
- Biconditional statement (if and only if ←→)
- Exclusive Or (XOR)

Negation

Let p be a proposition. The *negation of* p, denoted by $\neg p$ (also denoted by $\overline{p}$), is the statement

"It is not the case that p."

The proposition $\neg p$ is read "not p." The truth value of the negation of p, $\neg p$, is the opposite of the truth value of p.

TABLE 1 The Truth Table for the Negation of a Proposition.

p	$\neg p$
T	F
F	T

Example

- "Today is Friday"
  - It is not the case that today is Friday
  - Today is not Friday
- "Michaels PC runs Linux"
  - It is not the case that Michaels PC runs Linux
  - Michaels PC does not run Linux

Conjunction

Let p and q be propositions. The *conjunction* of p and q, denoted by $p \wedge q$, is the proposition "p and q." The conjunction $p \wedge q$ is true when both p and q are true and is false otherwise.

TABLE 2 The Truth Table for the Conjunction of Two Propositions.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Example

- p: “Today is Friday”, q: “It is raining today”
- p ∧ q “Today is Friday and it is raining today”
  - true: on rainy Fridays
  - false otherwise:
    - Any day that is not a Friday
    - Fridays when it does not rain

Disjunction

Let p and q be propositions. The *disjunction* of p and q, denoted by $p \vee q$, is the proposition "p or q." The disjunction $p \vee q$ is false when both p and q are false and is true otherwise.

TABLE 3 The Truth Table for the Disjunction of Two Propositions.

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Example

- p ∨ q: “Today is Friday or it is raining today”
  - True:
    - Today is Friday
    - It is raining today
    - It is a rainy Friday
  - False
    - Today is not Friday and it does not rain

Exclusive Or

Let p and q be propositions. The *exclusive or* of p and q, denoted by $p \oplus q$, is the proposition that is true when exactly one of p and q is true and is false otherwise.

TABLE 4 The Truth Table for the Exclusive Or of Two Propositions.

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

Conditional Statement

Let p and q be propositions. The *conditional statement* $p \rightarrow q$ is the proposition "if p, then q." The conditional statement $p \rightarrow q$ is false when p is true and q is false, and true otherwise. In the conditional statement $p \rightarrow q$, p is called the *hypothesis* (or *antecedent* or *premise*) and q is called the *conclusion* (or *consequence*).

TABLE 5 The Truth Table for the Conditional Statement $p \rightarrow q$.

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Conditional statement p→q

if p, then q if p, q p is sufficient for q q if p q when p a necessary condition for p is q q is unless ┐ p	p implies q p only if q a sufficient condition for q is p q whenever p q is necessary for p q follows from p

Example
If I am elected I will lower taxes.

TABLE 5 The Truth Table for the Conditional Statement $p \to q$.

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

$p \rightarrow q$

- p only if q:
  - p cannot be true when q is not true
  - The statement is false if p is true but q is false
  - When p is false, q may be either true or false
  - Not to use "q only if p" to express $p \rightarrow q$
- q unless $\neg$ p
  - If $\neg$ p is false, then q must be true
  - The statement is false when p is true but q is false, but the statement is true otherwise

TABLE 5 The Truth Table for the Conditional Statement $p \rightarrow q$.

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Example

- "If today is Friday, then 2+3=6"
  - The statement is true every day except Friday even though 2+3=6 is false

TABLE 5 The Truth Table for the Conditional Statement $p \rightarrow q$.

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Biconditional Statement

Let p and q be propositions. The *biconditional statement* $p \leftrightarrow q$ is the proposition "p if and only if q." The biconditional statement $p \leftrightarrow q$ is true when p and q have the same truth values, and is false otherwise. Biconditional statements are also called *bi-implications*.

TABLE 6 The Truth Table for the Biconditional $p \leftrightarrow q$.

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Example

- P: “you can take the flight”, q: “you buy a ticket”
- P ←→ q: “You can take the flight if and only if you buy a ticket”
  - This statement is true
    - If you buy a ticket and take the flight
    - If you do not buy a ticket and you cannot take the flight
- Note that P ←→ q has exactly the same truth values as (p->q) ^ (q->p)

Truth Table of Compound Preposition

TABLE 7 The Truth Table of $(p \vee \neg q) \rightarrow (p \wedge q)$.

p	q	$\neg q$	$p \vee \neg q$	$p \wedge q$	$(p \vee \neg q) \rightarrow (p \wedge q)$
T	T	F	T	T	T
T	F	T	T	F	F
F	T	F	F	F	T
F	F	T	T	F	F

Precedence of Logical Operators

TABLE 8
Precedence of Logical Operators.

Operator	*Precedence*
$\neg$	1
$\wedge$ $\vee$	2 3
$\rightarrow$ $\leftrightarrow$	4 5

Bit Operations

TABLE 9 Table for the Bit Operators *OR*, *AND*, and *XOR*.

x	y	$x \vee y$	$x \wedge y$	$x \oplus y$
0	0	0	0	0
0	1	1	0	1
1	0	1	0	1
1	1	1	1	0

Example

Find the bitwise OR, AND, XOR of the bit strings 01 1011 0110 and 11 0001 1101

Solution:

01 1011 0110	
11 0001 1101	
11 1011 1111	bitwise *OR*
01 0001 0100	bitwise *AND*
10 1010 1011	bitwise *XOR*