Ceng 124
Discrete Structures

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Topics

- 9.1 Relations
- 9.2 n-ary Relations

Topics

- 9.1 Relations and Their Properties
  - Relations:
    - Reflexive
    - Symmetric and Antisymmetric
    - Transitive
    - Combining Relations
- 9.2 n-ary Relations and Their Properties
  - n-ary relations
  - Databases and Relations
  - Operations on n-ary Relations

Introduction

- Relationship between a program and its variables.
- Integers that are congruent modulo k.
- Pairs of cities linked by airline flights in a network.

Relations and Their Properties

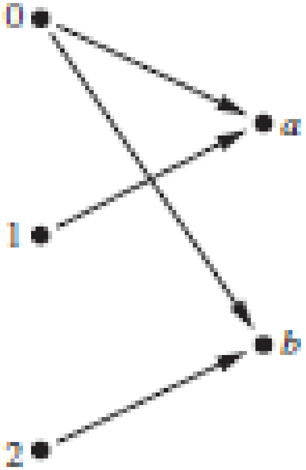
- If we want to describe a relationship between elements of two sets A and B, we can use ordered pairs with their first element taken from A and their second element taken from B.
- Since this is a relation between two sets, it is called a binary relation.
- Definition: Let A and B be sets. A binary relation from A to B is a subset of A x B.

Relations and Their Properties (cont.)

- Notation:

  $aRb \Leftrightarrow (a, b) \in R$

  $a \not R b \Leftrightarrow (a, b) \notin R$



R	a	b
0	×	×
1	×	
2		×

Relations and Their Properties (cont.)

- Example:

  A = set of all districts

  B = set of the all cities in the Turkey.

  Define the relation R by specifying that (a, b)

  belongs to R if district a is in city b.

  (Etimesgut, Ankara)

  (Alanya , Antalya) → are in R.

  (Üsküdar, İstanbul)

Functions as Relations

- The graph of a function f is the set of ordered pairs (a, b) such that b = f(a)
- The graph of f is a subset of A x B $\Rightarrow$ it is a relation from A to B
- Conversely, if R is a relation from A to B such that every element in A is the first element of exactly one ordered pair of R, then a function can be defined with R as its graph.

Relations on a Set

- Definition: A relation on the set A is a relation from A to A.
- In other words, a relation on the set A is a subset of A x A.
- Example: Let A = {1, 2, 3, 4}. Which ordered pairs are in the relation

  R = {(a, b) | a divides b} ?
- Solution Since (a, b) is in R if and only if a and b are positive integers not exceeding 4 such that a divides b

  R = {(1,1), (1,2), (1,3), (1,4), (2,2), (2,4), (3,3), (4,4)}

Properties of Relations

- Reflexive
- Symmetric
- Antisymmetric
- Transitive

Reflexive

- Definition: A relation R on a set A is called reflexive if $(a, a) \in R$ for every element $a \in A$.
- Example: Consider the following relations on {1, 2, 3, 4}.

  R1 = {(1,1), (1,2), (2,1), (2,2), (3,4), (4,1), (4,4)}
  R2 = {(1,1), (1,2), (2,1)}
  R3 = {(1,1), (1,2), (1,4), (2,1), (2,2), (3,3), (3,4), (4,1), (4,4)}
  R4 = {(2,1), (3,1), (3,2), (4,1), (4,2), (4,3)}
  R5 = {(1,1), (1,2), (1,3), (1,4), (2,2), (2,3), (2,4), (3,3), (3,4), (4,4)}
  R6 = {(3,4)}

  Which of these relations are reflexive?

Reflexive (cont.)

- Solution: R3 and R5: reflexive

  both contain all pairs of the form (a, a): (1,1), (2,2), (3,3) (4,4).

- R1, R2, R4 and R6: not reflexive

  not contain all of these ordered pairs. (3,3) is not in any of these relations.

Symmetric – Antisymmetric

- Definitions:
- A relation R on a set A is called symmetric if $(b, a) \in R$ whenever $(a, b) \in R$, for all $a, b \in A$.
- A relation R on a set A such that $(a, b) \in R$ and $(b, a) \in R$ only if $a = b$, for all $a, b \in A$, is called antisymmetric.

Example

- Consider the following relations on {1,2,3,4}. Which of the realtions are symmetric and which are antisymmetric?
- **R_1 = {(1,1), (1,2), (2,1), (2,2), (3,4), (4,1), (4,4)}**
  R_2 = {(1,1), (1,2), (2,1)}
  R_3 = {(1,1), (1,2), (1,4), (2,1), (2,2), (3,3), (4,1), (4,4)}
  R_4 = {(2,1), (3,1), (3,2), (4,1), (4,2), (4,3)}
  R_5 = {(1,1), (1,2), (1,3), (1,4), (2,2), (2,3), (2,4), (3,3), (3,4), (4,4)}
  R_6 = {(3,4)}
- R_2 , R_3: symmetric $\Leftarrow$ each case (b, a) belongs to the relation whenever (a, b) does.
  For R_2: only thing to check that both (1,2) , (2,1) belong to the relation
  For R_3: it is necessary to check that both (1,2) , (2,1) belong to the relation.
- R_4, R_5 and R_6: antisymmetric $\Leftarrow$for each of these relations there is no pair of elements a and b with
  $a \neq b$ such that both (a, b) and (b, a) belong to the relation.

Transitive

- Definition: A relation R on a set A is called transitive if whenever $(a, b) \in R$ and $(b,c) \in R$, then $(a, c) \in R$, for all $a, b, c \in R$.

Example

- Which of the following relations are transitive?
- **R_1 = {(1,1), (1,2), (2,1), (2,2), (3,4), (4,1), (4,4)}**
  R_2 = {(1,1), (1,2), (2,1)}
  R_3 = {(1,1), (1,2), (1,4), (2,1), (2,2), (3,3), (4,1), (4,4)}
  R_4 = {(2,1), (3,1), (3,2), (4,1), (4,2), (4,3)}
  R_5 = {(1,1), (1,2), (1,3), (1,4), (2,2), (2,3), (2,4), (3,3), (3,4), (4,4)}
  R_6 = {(3,4)}
- verify that if (a, b) and (b, c) belong to this relation then (a, c) belongs also to the relation
- R_1 : not transitive $\Leftarrow$ (3,4) and (4,1) belong to R_1, but (3,1) does not.
- R_2 : not transitive $\Leftarrow$ (2,1) and (1,2) belong to R_2, but (2,2) does not.
- R_3 : not transitive $\Leftarrow$ (4,1) and (1,2) belong to R_3, but (4,2) does not.
- R_4 , R_5 , R_6 : transitive $\Leftarrow$ R_4 transitive since (3,2) and (2,1), (4,2) and (2,1), (4,3) and (3,1), and (4,3) and (3,2) are the only such sets of pairs, and (3,1) , (4,1) and (4,2) belong to R_4. Same reasoning for R_5 and R_6.

Combing Relations

- Let A = {1, 2, 3} and B = {1, 2, 3, 4, }. The relations

  R_1 = {(1,1), (2,2), (3,3)} and
  R_2 = {(1,1), (1,2), (1,3), (1,4)} can be combined to obtain:

  $R_1 \cup R_2 = \{(1,1), (1,2), (1,3), (1,4), (2,2), (3,3)\}$

  $R_1 \cap R_2 = \{(1,1)\}$

  $R_1 - R_2 = \{(2,2), (3,3)\}$
  $R_2 - R_1 = \{(1,2), (1,3), (1,4)\}$

9.2 n-ary Relations and Their Properties

- Relationship among elements of more than 2 sets often arise: n-ary relations.
- There is a relationship involving the name of a student, student major, student grade point average.
- There is a relationship involving the airline, flight number, starting point, destination, departure time, arrival time.

n-ary Relations

- Definition: Let A_1, A_2, …, A_n be sets. An n-ary relation on these sets is a subset of A_1 x A_2 x….x A_n where A_i are the domains of the relation, and n is called its degree.
- Example: Let R be the relation on N x N x N consisting of triples (a, b, c) where a, b, and c are integers with $a<b<c$. Then $(1,2,3) \in R$, but $(2,4,3) \notin R$. The degree of this relation is 3. Its domains are equal to the set of integers.

Databases and Relations

- Relational database model has been developed for information processing
- A database consists of records, which are n-tuples made up of fields

  The fields contains information such as: Name, Student #, Major, Grade
- The relational database model represents a database of records or n-ary relation
- The relation is R (Student-Name, Id-number, Major, GPA)

Examples of Records

- (Ackermann, 231455, Computer Science, 3.88)

  (Adams, 888323, Physics, 3.45)

  (Chou, 102147, Computer Science, 3.49)

  (Goodfriend, 453876, Mathematics, 3.45)

  (Rao, 678543, Mathematics, 3.90)

  (Stevens, 786576, Psychology, 2.99)

TABLE 1 **Students.**

Student_name	*ID_number*	*Major*	*GPA*
Ackermann	231455	Computer Science	3.88
Adams	888323	Physics	3.45
Chou	102147	Computer Science	3.49
Goodfriend	453876	Mathematics	3.45
Rao	678543	Mathematics	3.90
Stevens	786576	Psychology	2.99

- Each column of the table corresponds to an *attribute* of the database.
- A domain of an *n*-ary relation is called a ***primary key*** when the value of the *n*-tuple from this domain determines the *n*-tuple.

Operations on n-ary Relations

- **Selection Operator:** Let R be an n-ary relation and C a condition that elements in R may satisfy. Then the selection operator Sc maps n-ary relation R to the n-ary relation of all n-tuples from R that satisfy the condition C.

Example

- To find the records, the condition Major="Computer Science" The result is the two 4-tuples:

  (Ackermann, 231455, Computer Science, 3.88)

  (Chou, 102147, Computer Science,3.49).

- To find the records, the condition GPA > 3.5. The result is the two 4-tuples:

  (Ackermann, 231455, Computer Science, 3.88)

  (Rao, 678543, Mathematics, 3.90).

- To find the records, the condition (Major="Computer Science" ∧ GPA > 3.5). The result consists of the single 4-tuple:

  (Ackermann, 231455, Computer Science, 3.88).

TABLE 1 Students.

Student_name	*ID_number*	*Major*	*GPA*
Ackermann	231455	Computer Science	3.88
Adams	888323	Physics	3.45
Chou	102147	Computer Science	3.49
Goodfriend	453876	Mathematics	3.45
Rao	678543	Mathematics	3.90
Stevens	786576	Psychology	2.99

Operations on n-ary Relations (cont.)

- Projection: The projection $P_{i_1, i_2, \ldots, i_m}$ maps the n-tuple (a1, a2, …, an) to the m-tuple $(a_{i_1}, a_{i_2}, \ldots, a_{i_m})$ where m <= n.

Example

- What relation results when the projection *P*1,4 is applied to the relation in Table 1?
- Solution: When the projection P1,4 is used, the second and third columns of the table are deleted, and pairs representing student names and grade point averages are obtained. Table 2 displays the results of this projection.

TABLE 1 Students.

Student_name	*ID_number*	*Major*	*GPA*
Ackermann	231455	Computer Science	3.88
Adams	888323	Physics	3.45
Chou	102147	Computer Science	3.49
Goodfriend	453876	Mathematics	3.45
Rao	678543	Mathematics	3.90
Stevens	786576	Psychology	2.99

TABLE 2 GPAs.

Student_name	*GPA*
Ackermann	3.88
Adams	3.45
Chou	3.49
Goodfriend	3.45
Rao	3.90
Stevens	2.99

Operations on n-ary Relations (cont.)

- Join: Let R be a relation of degree m and S a relation of degree n. The join Jp(R,S), where p <= m and p <= n, is a relation of degree m + n – p that consists of all (m + n – p)-tuples (a1, a2, …, am-p, c1, c2, …, cp, b1, b2, …, bn-p), where the m-tuple (a1, a2, …, am-p, c1, c2, …, cp) belongs to R and the n-tuple (c1, c2, …, cp, b1, b2, …, bn-p) belongs to S.

Example

- What relation results when the join operator J2 is used to combine the relation displayed in Tables 5 and 6?

TABLE 5 Teaching_assignments.

Professor	*Department*	*Course_ number*
Cruz	Zoology	335
Cruz	Zoology	412
Farber	Psychology	501
Farber	Psychology	617
Grammer	Physics	544
Grammer	Physics	551
Rosen	Computer Science	518
Rosen	Mathematics	575

TABLE 6 Class_schedule.

Department	*Course_ number*	*Room*	*Time*
Computer Science	518	N521	2:00 P.M.
Mathematics	575	N502	3:00 P.M.
Mathematics	611	N521	4:00 P.M.
Physics	544	B505	4:00 P.M.
Psychology	501	A100	3:00 P.M.
Psychology	617	A110	11:00 A.M.
Zoology	335	A100	9:00 A.M.
Zoology	412	A100	8:00 A.M.

Solution

The join *J*2 produces the relation shown in Table 7.

TABLE 7 Teaching_schedule.

Professor	*Department*	*Course_number*	*Room*	*Time*
Cruz	Zoology	335	A100	9:00 A.M.
Cruz	Zoology	412	A100	8:00 A.M.
Farber	Psychology	501	A100	3:00 P.M.
Farber	Psychology	617	A110	11:00 A.M.
Grammer	Physics	544	B505	4:00 P.M.
Rosen	Computer Science	518	N521	2:00 P.M.
Rosen	Mathematics	575	N502	3:00 P.M.

SQL-n-ary Relations

- The SQL statement

  SELECT Professor, Time

  FROM Teaching_assignments, Class_schedule

  WHERE Department='Mathematics'

- is used to find the projection P1,5 of the 5-tuples in the database (shown in Table 7), which is the join J2 of the Teaching_assignments and Class_schedule databases in Tables 5 and 6, respectively,
- which satisfy the condition: Department = Mathematics. The output would consist of the single 2-tuple (Rosen, 3:00 p.m.). The SQL FROM clause is used here to find the join of two different databases.